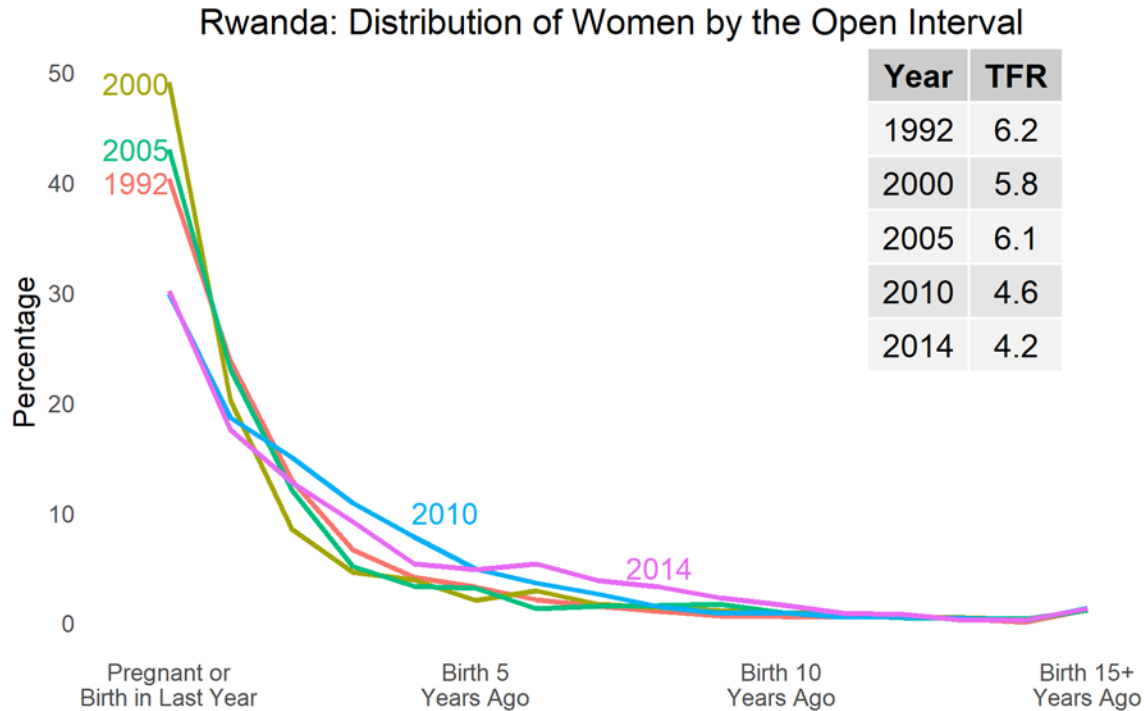


SUPPLEMENT 4. Curve-Fitting of the Open Interval Distribution

In early tabulations we noticed that the shape of the open interval distribution was markedly similar across countries. The married women were clustered close to the shorter intervals since their last birth, with regularly descending percentages toward the longer intervals. We therefore questioned whether a simple fitted curve with only a few parameters might be possible.

Using all 15 intervals (from the first year to 15 years) we generated curves like those shown in the following figure for Rwanda at various dates.



(Note that the 2010 and 2014 lines start at identical points, for 30%).

The dominant pattern in all other countries is the same. Women are concentrated to the left. Then the curve descends, at varying slopes depending upon the prevailing fertility level, as shown for the TFR values in the Rwanda figure. The descent is due partly to fewer women at the upper ages, where the open interval is longer.

Because this pattern is so universal, we fitted various equations to the typical shape, and found a “power” equation to provide a very close fit. It is of the form $Y = “a” \text{ times } (X \text{ taken to the “b” power})$ or $Y=a(x^b)$. It is a negative exponential curve as in the Figure, in which the curve falls off faster with a larger “b” value. The curve as a whole is placed higher or lower according to the “a” value.

A close relation exists between the open interval distribution and the usual fertility measures of the GFR and the TFR. The following Matrix (“r” values, which give R2 values if squared) shows the correlations to be above 0.90 across the 74 countries with the latest surveys.

Matrix for Correlations Between Fertility and Open Interval

| | "a" Value | "b" Value | GFR | TFR |
|-----------|--------------|--------------|------|------|
| "a" Value | 1.00 | | | |
| "b" Value | 0.98 | 1.00 | | |
| GFR | 0.92 | 0.94 | 1.00 | |
| TFR | 0.93 | 0.95 | 0.99 | 1.00 |

(The matrix gives "r" values, which give R2 values when squared)

Trends: Over time the average values have moved toward lower fertility levels. The following table (bottom row) shows the percentage decline of 16% to 17% in the GFR and TFR in the 56 countries with multiple surveys, over an average period of 17 years. Changes in the "a" and "b" values, at 13% and 11%, have been basically similar. The "a" value estimates the proportion of women in the first interval, i.e. the level at the start of the curve. The decline was from about 42% to about 37% of women in that interval (first column). The change in the "b" value, indicates a softening in the slope of the curve, with fewer women at the start and more toward the longest intervals. (The R2 values hardly changed at all, most being well above 0.900.)

| | "a" Value | "b" Value | R2 | GFR | TFR |
|------------------|-----------|-----------|---------|--------|--------|
| Earliest Surveys | 0.416 | (1.287) | 0.949 | 165 | 4.8 |
| Latest Surveys | 0.369 | (1.147) | 0.944 | 140 | 4.0 |
| Change | (0.047) | 0.140 | (0.004) | (25) | (0.8) |
| % Change | (12.6) | 10.6 | (0.2) | (15.7) | (16.8) |

The search for summary measures of the full 15-year distribution of intervals can use the "a" and "b" values, along with such others as the percentage of intervals less than one year, two years, etc. Succinct summary measures can be employed to examine such determinants as age, family size, and residence, along with contraceptive use, intention to use, and unmet need. Separately, a study of the rapidity of change within individual countries is of interest.

The two summary measures, "a" and "b", appear in the following table; it pertains to the 74 countries used above and includes the latest surveys for those countries, plus 56 more for the earliest surveys in countries having multiple surveys, making 130 total. The table gives the R2 correlations between the "a" and "b" values, and also includes the general fertility rate (GFR) and the total fertility rate (TFR). Anyone can generate the full estimated curve along the X-axis of 15 years by plugging the "a" and "b" values into the equation for any country.

| Table for Curve Fitting: "a" and "b" Values, Their R2 Correlation, GFR and TFR | | | | | | | | | | | |
|---|-----------|-----------|-------|-----|-----|----------------------|-----------|-----------|-------|-----|-----|
| This table lists 130 surveys: 74 for the latest surveys in 74 countries, and 56 more for the earliest surveys in countries with multiple surveys. | | | | | | | | | | | |
| | "a" Value | "b" Value | R2 | GFR | TFR | | "a" Value | "b" Value | R2 | GFR | TFR |
| Afghanistan 2015 | 0.440 | -1.426 | 0.956 | 175 | 5.3 | Kyrgyz Republic 1997 | 0.229 | -0.82 | 0.917 | 118 | 3.4 |
| Albania 2008-09 | 0.065 | -0.132 | 0.390 | 46 | 1.6 | Kyrgyz Republic 2012 | 0.257 | -0.968 | 0.985 | 125 | 3.6 |
| Angola 2015-16 | 0.603 | -1.678 | 0.970 | 216 | 6.2 | Lesotho 2004 | 0.318 | -1.072 | 0.970 | 121 | 3.5 |
| Armenia 2000 | 0.079 | -0.251 | 0.672 | 56 | 1.7 | Lesotho 2014 | 0.312 | -1.05 | 0.929 | 118 | 3.3 |
| Armenia 2015-16 | 0.121 | -0.538 | 0.968 | 64 | 1.7 | Liberia 2007 | 0.412 | -1.303 | 0.958 | 181 | 5.2 |
| Azerbaijan 2006 | 0.089 | -0.328 | 0.600 | 66 | 2.0 | Liberia 2013 | 0.418 | -1.282 | 0.935 | 168 | 4.7 |
| Bangladesh 1999-00 | 0.333 | -1.076 | 0.918 | 130 | 3.3 | Madagascar 1992 | 0.455 | -1.483 | 0.979 | 212 | 6.1 |
| Bangladesh 2014 | 0.165 | -0.63 | 0.943 | 90 | 2.3 | Madagascar 2008-09 | 0.396 | -1.255 | 0.958 | 168 | 4.8 |
| Benin 1996 | 0.548 | -1.579 | 0.974 | 202 | 6.0 | Malawi 1992 | 0.470 | -1.514 | 0.956 | 223 | 6.7 |
| Benin 2011-12 | 0.508 | -1.475 | 0.956 | 175 | 4.9 | Malawi 2015-16 | 0.515 | -1.445 | 0.914 | 158 | 4.4 |
| Bolivia 1994 | 0.397 | -1.255 | 0.970 | 163 | 4.8 | Maldives 2009 | 0.228 | -0.827 | 0.962 | 82 | 2.5 |
| Bolivia 2008 | 0.281 | -0.963 | 0.913 | 121 | 3.5 | Mali 1995-96 | 0.529 | -1.585 | 0.979 | 234 | 6.7 |
| Brazil 1991 | 0.268 | -0.932 | 0.924 | 122 | 3.4 | Mali 2012-13 | 0.559 | -1.59 | 0.967 | 214 | 6.1 |
| Brazil 1996 | 0.147 | -0.538 | 0.931 | 89 | 2.5 | Moldova 2005 | 0.078 | -0.294 | 0.747 | 55 | 1.7 |
| Burkina Faso 1993 | 0.691 | -1.804 | 0.946 | 221 | 6.5 | Morocco 1992 | 0.412 | -1.294 | 0.981 | 127 | 4.0 |
| Burkina Faso 2010 | 0.581 | -1.616 | 0.961 | 206 | 6.0 | Morocco 2003-04 | 0.262 | -0.912 | 0.941 | 81 | 2.5 |
| Burundi 2010 | 0.732 | -1.942 | 0.971 | 203 | 6.4 | Mozambique 1997 | 0.440 | -1.407 | 0.991 | 183 | 5.2 |
| Burundi 2016-17 | 0.748 | -1.846 | 0.923 | 180 | 5.5 | Mozambique 2011 | 0.458 | -1.464 | 0.975 | 206 | 5.9 |
| Cambodia 2000 | 0.376 | -1.178 | 0.868 | 119 | 3.8 | Myanmar 2015-16 | 0.198 | -0.723 | 0.932 | 77 | 2.3 |
| Cambodia 2014 | 0.215 | -0.779 | 0.953 | 98 | 2.7 | Namibia 1992 | 0.410 | -1.325 | 0.957 | 176 | 5.4 |
| Cameroon 1991 | 0.437 | -1.417 | 0.972 | 208 | 5.8 | Namibia 2013 | 0.305 | -1.039 | 0.948 | 125 | 3.6 |

| | | | | | | | | | | | |
|-------------------------------|-------|--------|-------|-----|-----|-------------------------------|-------|--------|-------|-----|-----|
| Cameroon 2011 | 0.426 | -1.385 | 0.977 | 180 | 5.1 | Nepal 1996 | 0.320 | -1.097 | 0.976 | 167 | 4.6 |
| Central African Rep. 1994-95 | 0.428 | -1.388 | 0.966 | 183 | 5.1 | Nepal 2016 | 0.146 | -0.573 | 0.970 | 88 | 2.3 |
| Chad 1996-97 | 0.562 | -1.679 | 0.955 | 229 | 6.4 | Nicaragua 1998 | 0.284 | -0.976 | 0.973 | 132 | 3.6 |
| Chad 2014-15 | 0.597 | -1.697 | 0.954 | 230 | 6.4 | Nicaragua 2001 | 0.258 | -0.91 | 0.906 | 117 | 3.2 |
| Colombia 1990 | 0.245 | -0.875 | 0.956 | 105 | 2.8 | Niger 1992 | 0.566 | -1.687 | 0.978 | 250 | 7.0 |
| Colombia 2015 | 0.129 | -0.495 | 0.947 | 70 | 2.0 | Niger 2012 | 0.699 | -1.881 | 0.967 | 269 | 7.6 |
| Comoros 1996 | 0.414 | -1.349 | 0.937 | 148 | 4.6 | Nigeria 1990 | 0.561 | -1.615 | 0.934 | 207 | 6.0 |
| Comoros 2012 | 0.392 | -1.283 | 0.981 | 142 | 4.3 | Nigeria 2013 | 0.473 | -1.473 | 0.969 | 190 | 5.5 |
| Congo 2005 | 0.419 | -1.345 | 0.986 | 168 | 4.8 | Pakistan 1990-91 | 0.441 | -1.387 | 0.940 | 164 | 4.9 |
| Congo 2011-12 | 0.471 | -1.453 | 0.988 | 182 | 5.1 | Pakistan 2012-13 | 0.301 | -1.05 | 0.979 | 131 | 3.8 |
| Congo Democratic Rep. 2007 | 0.459 | -1.477 | 0.975 | 210 | 6.3 | Paraguay 1990 | 0.395 | -1.255 | 0.943 | 160 | 4.7 |
| Congo Democratic Rep. 2013-14 | 0.535 | -1.641 | 0.982 | 225 | 6.6 | Peru 1991-92 | 0.301 | -1.028 | 0.969 | 121 | 3.5 |
| Cote d'Ivoire 1994 | 0.571 | -1.595 | 0.949 | 187 | 5.3 | Peru 2012 | 0.198 | -0.746 | 0.911 | 86 | 2.6 |
| Cote d'Ivoire 2011-12 | 0.469 | -1.423 | 0.973 | 174 | 5.0 | Philippines 1993 | 0.302 | -1.041 | 0.985 | 138 | 4.1 |
| Dominican Rep. 1991 | 0.247 | -0.905 | 0.981 | 125 | 3.3 | Philippines 2013 | 0.225 | -0.824 | 0.940 | 101 | 3.0 |
| Dominican Rep. 2013 | 0.172 | -0.696 | 0.977 | 89 | 2.5 | Rwanda 1992 | 0.888 | -2.06 | 0.919 | 197 | 6.2 |
| Egypt 1992 | 0.353 | -1.136 | 0.959 | 136 | 3.9 | Rwanda 2014-15 | 0.646 | -1.629 | 0.832 | 142 | 4.2 |
| Egypt 2014 | 0.271 | -0.971 | 0.985 | 127 | 3.5 | Sao Tome and Principe 2008-09 | 0.457 | -1.392 | 0.952 | 164 | 4.9 |
| Ethiopia 2000 | 0.588 | -1.655 | 0.940 | 183 | 5.5 | Senegal 1992-93 | 0.504 | -1.536 | 0.974 | 202 | 6.0 |
| Ethiopia 2016 | 0.544 | -1.506 | 0.925 | 156 | 4.6 | Senegal 2016 | 0.602 | -1.607 | 0.934 | 156 | 4.7 |
| Gabon 2000 | 0.300 | -1.044 | 0.946 | 153 | 4.2 | Sierra Leone 2008 | 0.429 | -1.323 | 0.942 | 180 | 5.1 |
| Gabon 2012 | 0.359 | -1.225 | 0.947 | 144 | 4.1 | Sierra Leone 2013 | 0.489 | -1.428 | 0.938 | 169 | 4.9 |
| Gambia 2013 | 0.561 | -1.635 | 0.974 | 185 | 5.6 | South Africa 1998 | 0.195 | -0.735 | 0.894 | 100 | 2.9 |

| | | | | | | | | | | | |
|-------------------|-------|--------|-------|-----|-----|---------------------|-------|--------|-------|-----|-----|
| Ghana 1993 | 0.498 | -1.465 | 0.963 | 178 | 5.2 | Swaziland 2006-07 | 0.307 | -1.055 | 0.946 | 137 | 3.8 |
| Ghana 2014 | 0.386 | -1.242 | 0.977 | 143 | 4.2 | Tajikistan 2012 | 0.249 | -0.917 | 0.959 | 134 | 3.8 |
| Guatemala 1995 | 0.317 | -1.114 | 0.987 | 177 | 5.1 | Tanzania 1991-92 | 0.480 | -1.498 | 0.972 | 212 | 6.2 |
| Guatemala 2014-15 | 0.254 | -0.904 | 0.981 | 112 | 3.1 | Tanzania 2015-16 | 0.482 | -1.437 | 0.962 | 178 | 5.2 |
| Guinea 1999 | 0.490 | -1.448 | 0.963 | 195 | 5.5 | Timor-Leste 2009-10 | 0.575 | -1.579 | 0.923 | 175 | 5.7 |
| Guinea 2012 | 0.462 | -1.415 | 0.937 | 176 | 5.1 | Timor-Leste 2016 | 0.384 | -1.197 | 0.913 | 136 | 4.2 |
| Guyana 2009 | 0.152 | -0.602 | 0.914 | 94 | 2.8 | Togo 1998 | 0.520 | -1.513 | 0.965 | 175 | 5.2 |
| Haiti 1994-95 | 0.406 | -1.324 | 0.984 | 156 | 4.8 | Togo 2013-14 | 0.479 | -1.425 | 0.965 | 163 | 4.8 |
| Haiti 2012 | 0.326 | -1.086 | 0.943 | 117 | 3.5 | Turkey 1993 | 0.177 | -0.668 | 0.939 | 90 | 2.5 |
| Honduras 2005-06 | 0.290 | -0.991 | 0.923 | 117 | 3.3 | Turkey 2003 | 0.160 | -0.646 | 0.956 | 79 | 2.2 |
| Honduras 2011-12 | 0.235 | -0.844 | 0.974 | 107 | 2.9 | Uganda 1995 | 0.592 | -1.773 | 0.966 | 248 | 6.9 |
| India 1992-93 | 0.230 | -0.861 | 0.967 | 130 | 3.4 | Uganda 2016 | 0.559 | -1.568 | 0.966 | 189 | 5.4 |
| India 2015-16 | 0.125 | -0.52 | 0.968 | 81 | 2.2 | Ukraine 2007 | 0.066 | -0.254 | 0.798 | 39 | 1.2 |
| Indonesia 1991 | 0.256 | -0.906 | 0.929 | 108 | 3.0 | Uzbekistan 1996 | 0.259 | -0.9 | 0.959 | 123 | 3.3 |
| Indonesia 2012 | 0.185 | -0.698 | 0.914 | 88 | 2.6 | Vietnam 1997 | 0.184 | -0.645 | 0.876 | 80 | 2.3 |
| Jordan 1990 | 0.476 | -1.453 | 0.980 | 169 | 5.6 | Vietnam 2002 | 0.106 | -0.341 | 0.753 | 60 | 1.9 |
| Jordan 2012 | 0.326 | -1.08 | 0.967 | 112 | 3.5 | Yemen 2013 | 0.454 | -1.373 | 0.959 | 146 | 4.4 |
| Kazakhstan 1995 | 0.144 | -0.551 | 0.839 | 83 | 2.5 | Zambia 1992 | 0.495 | -1.557 | 0.971 | 219 | 6.5 |
| Kazakhstan 1999 | 0.105 | -0.351 | 0.819 | 67 | 2.0 | Zambia 2013-14 | 0.581 | -1.597 | 0.954 | 184 | 5.3 |
| Kenya 1993 | 0.423 | -1.305 | 0.962 | 182 | 5.4 | Zimbabwe 1994 | 0.463 | -1.373 | 0.951 | 148 | 4.3 |
| Kenya 2014 | 0.400 | -1.236 | 0.929 | 141 | 3.9 | Zimbabwe 2015 | 0.470 | -1.379 | 0.931 | 144 | 4.0 |